Fermi National Accelerator Laboratory

FERMILAB-Conf-87/78-T

DUAL AMPLITUDES AND MULTI-GLUON PROCESSES*

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Abstract

A new technique to calculate tree-level multi-gluon amplitudes is presented. The basic idea is to expand the full amplitude in terms of dual diagrams. Each dual diagram is the combination with proper weights of some set of Feynman diagrams. The dual amplitudes are invariant under cyclic permutations of the external legs and are gauge invariant. Powerful identities relate different non-cyclic permutations with one another, dramatically simplifying the calculation of the full amplitude. The factorization of the dual amplitudes on the two particle channels is explicit and guarantees the cancellation of the double poles for collinear singularities at the amplitude level. The color algebra 'factors' out and the sum over colors can be performed independently of the kinematics. Relatively compact analytic expressions for processes like 2 gluons→4 gluons can then be obtained.

*Talk delivered by M. M. at 'Les Rencontres de Physique de la Vallée d'Aoste', La Thuile, Italy, March 1-7, 1987.

†Fermilab is operated by Universities Research Association, Inc., under contract with the United States Department of Energy.

1 The Dual Expansion

The calculation of multi-gluon amplitudes is one of the most challenging technical problems in perturbative QCD. Many tools have been developed trying to get simple analytic expressions for these kind of processes, but none seems to be the most appropriate yet. When calculating the Feynman diagrams contributing to a given reaction, many cancellations among the various terms are expected to occur on the basis of gauge invariance. A systematic procedure to efficiently isolate gauge invariant subsets of diagrams, however, has always been missing.

In this talk I will present a new technique that was recently developed in collaboration with S. Parke and Z. Xu [1], and that to our opinion provides a very efficient way of carrying out these calculations. The idea for this work came from the proposal, made by Parke and Taylor [2], that to the leading order in the $1/N$ expansion the amplitude squared for the scattering of two positive-helicity gluons into $n-2$ positive-helicity gluons takes the form [3]:

$$|\mathcal{M}_n(--+++\ldots)|^2 = c_n(g,N)(p_1\cdot p_2)^4 \sum_{perm} \frac{1}{(p_1\cdot p_2)(p_2\cdot p_3)\ldots(p_n\cdot p_1)}, \tag{1}$$

where all the gluons are taken as outgoing, $c_n(g,N) = g^{2n-4}N^{n-2}(N^2-1)/2^{n-4}$ and the sum is over all the *non-cyclic* permutations of $1,2,\ldots,n$.

It seems natural to interpret this expression as the square of some dual amplitude. The fact that the amplitude squared takes such a simple form if written in this fashion suggests that finding a procedure to make this duality manifest at the matrix element level may significantly simplify the calculations. This is the idea we tried to pursue. I will now introduce the concept of *dual perturbation theory*, pointing out some of its most remarkable properties relevant to this problem.

Dual perturbation theory [4] is built out of dual diagrams. A dual diagram (fig.1) is invariant under cyclic permutations of the external legs, and the full amplitude for the scattering of n particles is given by:

$$\mathcal{A}_n = \sum_{perm} tr\,(\lambda_1\lambda_2\ldots\lambda_n)\; A(1,2,\ldots,n). \tag{2}$$

$A(1,2,\ldots,n)$ represents the amplitude corresponding to the dual diagram with the ordering $(1,2,\ldots,n)$ of the external legs. The λ matrices are the matrices of the symmetry group in the fundamental representation. We will choose our gauge group to be $SU(N)$, and the external particles will transform as the adjoint representation of $SU(N)$.

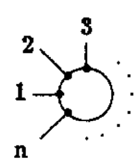

Figure 1: A dual diagram.

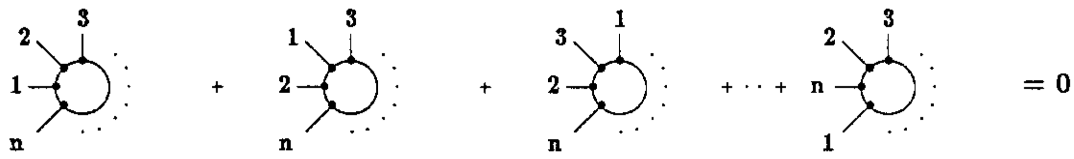

Figure 2: Dual Ward-Identity.

Probably the best way to think of dual diagrams and dual amplitudes is to think in terms of strings [5]. Each dual diagram is then represented by a string diagram, and the amplitude can be obtained by using the usual Koba-Nielsen formula. The traces of λ matrices are just the Chan-Paton factors. The fundamental property of these dual amplitudes is that in the limit where the string tension (or, if you prefer, the Regge slope) goes to zero, $\mathcal{A}_n$ reproduces the Yang-Mills amplitude. This implies that a dual representation for a multi-gluon amplitude always exists.

Independently of the zero-slope limit being taken, a dual amplitude enjoys remarkable properties. These properties suggest that a Yang-Mills amplitude expressed in terms of dual amplitudes will assume a particularly simple form.

First of all $A(1,2,\ldots,n)$ is gauge invariant. This means that for *each* external gluon within *each* dual diagram that forms the full amplitude we can choose a suitable parametrization of the polarization. Hence, breaking up the full amplitude in dual amplitudes gives rise to a systematic procedure for identifying gauge invariant subsets of diagrams. The gauge invariance allows us to obtain the dual amplitude corresponding to a non-cyclic permutation of $(1,2,\ldots,n)$ that mixes same-helicity gluons by just permuting the indices of the momenta that are contained in the amplitude. This saves us from having to calculate all of the $(n-1)!/2$ different dual diagrams that contribute to $\mathcal{A}_n$. Some of the permutations that interchange gluons with different helicities may be obtained through the Ward identity graphically represented in fig.2. The dual diagrams do not contain the color factors, so it would be very difficult to recover this identity in the Feynman diagram expansion.

The dual expansion, furthermore, gives rise to the following representation for the amplitude squared:

$$\sum_{colors} |\mathcal{A}_n|^2 = \frac{N^{n-2}(N^2-1)}{2^n}\left\{\sum_{perm} |A(1,2,\ldots,n)|^2 + \mathcal{O}(N^{-2})\right\}. \tag{3}$$

The leading terms in the expansion come from the square of the single dual amplitudes corresponding to the various non-cyclic permutations, while the $\mathcal{O}(N^{-2})$ terms represent the interference among different permutations. The fact that the non-leading terms come from the interference of dual diagrams with different orderings of the external legs can be easily understood by realizing that these interferences can be represented by non-planar diagrams[1]. It is clear that the Parke and Taylor expression (eq.(1)) has

[1]Strictly speaking the planar diagrams (*i.e.* the squares of the dual amplitudes) generate the leading term (N^n) together with subleading terms, of the order N^{n-2} at least. The most important terms in the interference will then conspire with the $\mathcal{O}(N^{n-2})$ terms in the sum of squares to give rise to the structure exhibited in equation (3). Therefore, what I defined *interference* above is actually a mixture of true interference terms with subleading incoherent terms.

the form of equation (3). What is remarkable in this representation of the amplitude squared is the fact that the leading part is a sum of squares. This has a very important immediate consequence: any pole that must cancel in the full amplitude squared will have to be cancelled, at the matrix element level, within each dual diagram. If it were not so, at the order N^n the cancellation could not occur.

The best example of this is the case of collinear singularities. In calculating the Feynman diagrams for a certain process, propagators give rise to simple poles in the kinematical invariants $s_{ij} = (p_i + p_j)^2$. When squaring the amplitude, one would then naively expect double poles in s_{ij} to appear. However we know that these singularities are forbidden, only simple poles for collinear gluons are allowed [6]. In general this property is rather obscure at the matrix element level, and becomes apparent only after taking the square. Here many cancellations occur among terms from various Feynman diagrams, and quite mysteriously they conspire to fully cancel the double poles. My previous argument indicates however that if we were calculating the amplitude by using a dual expansion, these cancellations should be explicit already at the matrix element level. This suggests that the final expression for the amplitude will be much more simple than in general, because of the number of terms that we know in advance will have to disappear.

The last point that I want to make before explaining how to carry out this program is the following: the dual expansion gives a systematic procedure to calculate amplitudes to the leading order in N. For practical purposes, where the group is $SU(3)$, the subleading part contributes only a few percent of the complete result. This is comparable with the contribution of the diagrams of the next order in α_{strong}, and can be neglected unless the radiative corrections are calculated too. This would simplify the task of calculating the square, because to the leading order in N only the sum of the single dual amplitudes squared is needed, the interference being suppressed.

2 Reconstructing Feynman Diagrams

One possible way to implement this procedure is to directly use the Koba-Nielsen representation of the amplitude in the zero-slope limit. It turns out, however, that when the number of gluons exceeds five the integration over the Koba-Nielsen variables is quite hard, and the expansion of the exponent containing the polarizations generates too many terms to make this route appealing. As an alternative, we decide to reorganize the Feynman diagram expansion in such a way as to make it look like the zero-slope limit of the dual expansion. In the zero-slope limit a dual amplitude gives rise to a sum of terms that exhibit the poles one would expect from the propagators present in the Feynman diagrams. It is possible to interpret these terms as portions of field theory Feynman diagrams. As an example we may take the four-gluon process (fig.3). $A(1,2,3,4)$, which in this case is nothing but the Veneziano amplitude, exhibits an s-pole and a t-pole singularity in the zero-slope limit. When these terms are combined with those arising from the two additional permutations (1,3,2,4) and (2,1,3,4), the sum of the three Feynman diagrams corresponding to the s,t and u poles is reproduced. Of course the non-singular contact interaction is generated as well. Our suggestion is to follow this route in the opposite direction: we write down all the possible Feynman diagrams relevant to a certain process and we isolate those that would appear in the zero-slope limit of the dual diagram corresponding to some ordering. Then we sum up with the proper weights the Feynman diagrams in this group, obtaining the dual amplitude

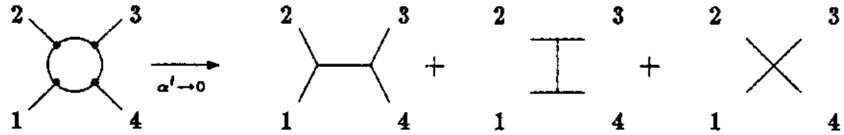

Figure 3: The zero-slope limit of a dual diagram.

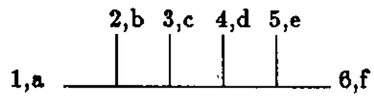

Figure 4: A six-gluon diagram.

corresponding to the given ordering. It is our claim that this object will be particularly simple to calculate and will give rise to a compact expression for the full amplitude.

To find which Feynman diagrams will contribute to a given dual diagram it is sufficient to study the color structure of the diagrams. As an example, take the six-gluon diagram in fig.4. The indices $a,b,..,f$ are color indices. The color structure of the diagram is given by $\sum_{X,Y,Z} f^{abX} f^{XcY} f^{YdZ} f^{Zef}$. It is easy to expand this factor in terms of λ matrices, obtaining the following result:

$$\sum_{X,Y,Z} f^{abX} f^{XcY} f^{YdZ} f^{Zef} = tr\ [[\lambda^a,\lambda^b],\lambda^c][\lambda^d,[\lambda^e,\lambda^f]]\ +\ tr\ [[\lambda^f,\lambda^e],\lambda^d][\lambda^c,[\lambda^b,\lambda^a]]. \tag{4}$$

These are exactly the color factors that appear in the dual expansion. The commutators of λ matrices can be expanded, and this particular Feynman diagram will then contribute to all the dual diagrams whose ordering appears in the expansion. The trace with the reversed ordering accounts for the dual diagrams with the reverse ordering. The plus sign is consistent with the fact that $A(1,2,\ldots,n) = (-)^n A(n,n-1,\ldots,1)$. In conclusion, to find the Feynman diagrams that contribute to a given dual diagram we just have to isolate the diagrams whose color factors contain a trace of λ matrices in the desired order. This procedure can be systematically followed for each n.

3 Five- and Six-Gluon Processes

In this section I will give a few examples of how this technique works. We choose to represent the gluon polarization with a pair of Weyl spinors, as recently proposed in [7]. This choice represents an improvement of the representation first introduced in [8]. Following this reference we employ the helicity amplitude formalism, *i.e.* we calculate amplitudes with given fixed external helicities. At the end we will sum over all the admissible helicity configurations.

We define the *kets* $|p\pm\rangle$ to be massless spinors with momentum p and helicity $\pm$ and the *bras* $\langle q\pm|$ to be the dual of a massless spinor with momentum q and helicity $\pm$. In this way the two helicity eigenstates

Figure 5: In a proper gauge, these are the only diagrams contributing to the dual amplitude $A(1^-,2^-,3^+,4^+,5^+)$.

of a gluon with momentum p are given by:

$$\epsilon_+^\mu(p) = \frac{\langle q-|\gamma^\mu|p-\rangle}{\sqrt{2}\langle qp\rangle}, \qquad \epsilon_-^\mu(p) = \frac{\langle q+|\gamma^\mu|p+\rangle}{\sqrt{2}\langle qp\rangle^*} \tag{5}$$

The *spinor product* $\langle qp\rangle$ is the scalar quantity obtained by multiplying $\langle q-|$ with $|p+\rangle$. The momentum q is arbitrary, provided it satisfies $q^2 = 0$ and $q \cdot p \neq 0$. This freedom in choosing what we will call the *reference momentum* stems from the gauge invariance of the polarization. Within a dual diagram, which is gauge invariant, we can assign a different reference momentum to each external gluon.
The only property of the spinor products that I want to recall here is the following: $|\langle qp\rangle|^2 = 2(p\cdot q)$. In this sense, up to a phase, a spinor product has to be thought of as the square root of the invariant s_{qp}. Many other properties of the spinor products[7], mainly due to Fierz identities, are very useful in simplifying the calculations.

As a first example of how this technique works, let me study the five-gluon amplitude [9]. There is only one independent helicity amplitude that contributes, the one with three positive and two negative helicities. Let us start by calculating the dual diagram with the ordering $(1^-,2^-,3^+,4^+,5^+)$. The obvious notation indicates the ordering of momenta around the loop and the helicity of the gluon with the given momentum. We will choose the reference momentum of the gluons 1 and 2 to be p_5, and the reference momentum of the gluons 3, 4 and 5 to be p_1. This is allowed by gauge invariance, and amounts to a gauge choice. It is easy to see that in this gauge all of the Feynman diagrams contributing to this process with the four-gluon contact interaction vanish. This is because the only products of polarizations that do not vanish are $\epsilon_-(2)\cdot\epsilon_+(3)$ and $\epsilon_-(2)\cdot\epsilon_+(4)$, as is easy to check by using the proper Fierz identities. As a consequence of this, also all the diagrams where the gluon 1 and the gluon 5 are attached to the same vertex can be easily seen to vanish. By using the above prescription for isolating the Feynman diagrams contributing to this dual diagram, it is easy to verify that the only Feynman diagrams that are relevant are the three given in figure 5. In the given gauge, the calculation of these three diagrams is quite easy, and the result is:

$$A(1^-,2^-,3^+,4^+,5^+) \;=\; (2)^{5/2}\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}. \tag{6}$$

From now on I will use the shorthand notation $\langle p_i p_j\rangle = \langle ij\rangle$ and I will put $g = 1$. Notice the simplicity of this expression: there is just one term left, out of the many hundreds that are generated by the Feynman rules. Notice also that the cancellation of the double poles in correspondence of collinear gluons is explicit at the amplitude level, as anticipated. The Altarelli-Parisi behaviour of the amplitude when two gluons are taken parallel is also explicit in this amplitude, as is easy to check. The origin of this behaviour stems from the factorization properties of the dual amplitudes.

By using gauge invariance it is straightforward to prove that the amplitudes corresponding to permutations of (1,2,3,4,5) that mix gluons with the same helicity are just obtained by permuting the momenta inside the expression (6). The other permutations can be obtained by using the dual Ward identity (fig.2). The result is:

$$A(i_1,i_2,i_3,i_4,i_5) \;=\; (2)^{5/2}\frac{\langle 12\rangle^4}{\langle i_1i_2\rangle\langle i_2i_3\rangle\langle i_3i_4\rangle\langle i_4i_5\rangle\langle i_5i_1\rangle}. \tag{7}$$

Equations (6) and (7) are sufficient to write the full amplitude $\mathcal{A}_5$, with the color factors given by the Chan-Paton prescription. It is interesting to note that the sum over colors of the amplitude squared can be performed independently of the explicit form of the amplitude. In fact, by just using the dual representation of the amplitude (eq.(2)) and the dual Ward identity it is possible to verify that:

$$\sum_{colors}|\sum_{perm} tr(\lambda_1\ldots\lambda_5)A(1,\ldots,5)|^2 \;=\; \frac{N^3(N^2-1)}{32}\sum_{perm}|A(1,\ldots,5)|^2, \tag{8}$$

This holds for every expression $A(1,\ldots,5)$ that is invariant under cyclic permutations, changes sign if the ordering is reversed, and satisfies the dual Ward identity. The partial sum over permutations that reverse the ordering — $(1,2,\ldots,n)\rightarrow(n,n-1,\ldots1)$ — is of course trivial, and just gives rise to an overall factor of 2.

By using equations (6) and (7) we can then obtain the standard expression for the amplitude squared:

$$\sum_{colors}|\mathcal{A}_5(--+++)|^2 \;=\; s_{12}^4\sum_{perm}\frac{1}{s_{12}s_{23}s_{34}s_{45}s_{51}}. \tag{9}$$

The sum over the helicities can be finally obtained by just replacing the overall factor s_{12}^4 with $\sum_{i,j=1}^5\, s_{ij}^4$ (no averaging performed yet).

I will now move to the six-gluon case [10]. Here we have two possible sets of helicities that contribute: $\mathcal{A}(--++++)$ and $\mathcal{A}(---+++)$. The first one is rather trivial, and it is the straightforward generalization of the five-gluon amplitude:

$$A(1-,2-,3+,4+,5+,6+) \;=\; 8i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 56\rangle\langle 61\rangle}. \tag{10}$$

Different permutations can be obtained as before by keeping fixed the numerator and permuting the momenta in the denominator.

The other amplitude is not as simple. It admits two different representations, one exhibiting the factorization on the two particle channels (*i.e.* the Altarelli-Parisi behaviour), and the other one exhibiting the factorization on the three particle channels. Here I will give the second one:

$$\begin{aligned}A(1^+,2^+,3^+,4^-,5^-,6^-) \;=\;& \frac{8i}{t_{234}^2s_{34}s_{61}}\frac{\langle 56\rangle[23]\langle 1+|\hat{2}+\hat{3}|4+\rangle^2}{\langle 23\rangle[56]}\\ &+\frac{8i}{t_{345}^2s_{34}s_{61}}\frac{\langle 45\rangle[12]\langle 3+|\hat{4}+\hat{5}|6+\rangle^2}{\langle 12\rangle[45]}\\ &-\frac{8i\,t_{123}}{s_{34}s_{61}}\frac{\langle 1+|\hat{2}+\hat{3}|4+\rangle\langle 3+|\hat{4}+\hat{5}|6+\rangle}{\langle 12\rangle\langle 23\rangle[45][56]}.\end{aligned} \tag{11}$$

I have used the following notation: $t_{ijk} = (p_i + p_j + p_k)^2$, $[ij] = \langle ij \rangle^*$ and $\hat{\imath} = p_i \cdot \gamma$. In this representation the cancellation of the redundant poles in the 3-4 and 6-1 channels is not explicit, but can be easily verified. Two more helicity configurations are needed:

$$A(1^+, 2^+, 4^-, 3^+, 5^-, 6^-) \quad \text{and} \quad A(1^+, 4^-, 2^+, 5^-, 3^+, 6-). \tag{12}$$

They can be obtained after some work by using the following Ward identities:

$$\begin{aligned} & A(1+,2+,3+,4-,5-,6-) + A(2+,1+,3+,4-,5-,6-) + A(2+,3+,1+,4-,5-,6-) \\ + \; & A(2+,3+,4-,1+,5-,6-) + A(2+,3+,4-,5-,1+,6-) \; = \; 0 \,, \end{aligned} \tag{13}$$

$$\begin{aligned} & A(2+,3+,4-,1+,5-,6-) + A(3+,2+,4-,1+,5-,6-) + A(3+,4-,2+,1+,5-,6-) \\ + \; & A(3+,4-,1+,2+,5-,6-) + A(3+,4-,1+,5-,2+,6-) \; = \; 0 \,. \end{aligned} \tag{14}$$

As in the five gluon case it is possible to perform the sum of the colors in the amplitude squared by just using duality and the Ward identities. The result is the following:

$$\begin{aligned} \sum_{colors} |\mathcal{A}_6|^2 \;\; = \;\; & \frac{N^2(N^2-1)}{64} \sum_{perm} \{ N^2 |A(1,2,3,4,5,6)|^2 + \\ & 2\, A^*(1,2,3,4,5,6)[A(1,3,5,2,6,4) + A(5,1,3,6,4,2) + A(3,5,1,4,2,6)]\}. \end{aligned} \tag{15}$$

This equation is obviously satisfied by both the helicity amplitudes, $\mathcal{A}(--++++)$ and $\mathcal{A}(---+++)$. Contrarily to the four and five gluon amplitudes, in this case the subleading terms do not vanish. However it is possible to show that they will not give rise to collinear singularities. In fact, by bringing two gluons parallel, say i and j, the full amplitude will behave as the product of a five gluon amplitude times the pole times the proper Altarelli-Parisi function $f(z)$. If the subleading part of the six-gluon amplitude were singular in s_{ij}, this would give rise to a non-leading contribution to the five-gluon process. Since there is no sub-leading term in the five-gluon amplitude, the sub-leading part of equation (15) is finite when s_{ij} goes to zero.

The sum over helicities can also be performed by just using the general properties of the amplitude. Let us define:

$$\begin{aligned} A_1(1,2,3,4,5,6) &= A(1^+, 2^+, 3^+, 4^-, 5^-, 6^-) \\ A_2(1,2,3,4,5,6) &= A(1^+, 2^+, 3^-, 4^+, 5^-, 6^-) \\ A_3(1,2,3,4,5,6) &= A(1^+, 2^-, 3^+, 4^-, 5^+, 6^-). \end{aligned} \tag{16}$$

It is then easy to check that the sum over colors and helicities of the square of $\mathcal{A}(+++---)$ is given by:

$$\begin{aligned} \sum_{colors} \sum_{helic.} |\mathcal{A}(+++---)|^2 = \frac{N^2(N^2-1)}{64} \sum_{all\,perm} \quad \{ \quad & H_1(1,2,3,4,5,6) + 2\, H_2(1,2,3,4,5,6) \\ + \; & \frac{1}{3} H_3(1,2,3,4,5,6) \; \}, \end{aligned} \tag{17}$$

where $H_i(1,2,3,4,5,6)$ is given by the expression within curly brackets in equation (15), with $A(1,2,\ldots,6)$ replaced by $A_i(1,2,\ldots,6)$. Notice that now the sum is over all the permutations, both cyclic *and* non-cyclic.

More details and the complete expression for the six-gluon amplitude will be contained in [1].

4 Conclusions

In conclusion, I have presented what we believe to be a very effective technique for calculating higher order processes in perturbative QCD. Using this technique we have gained a complete analytic control of amplitudes with up to six gluons. We do not expect the seven-gluon process to be much more complicated, because no new structures in the amplitude will appear until the eight-gluon case. In particular, the $A(--+++++)$ amplitudes will be the obvious extension of their five- and six-gluon equivalent. A long-term goal would be to find either recursive relations to generate higher point functions starting from lower order ones, or to find an alternative set of Feynman rules that embody the duality structure. This might follow from a more accurate study of the zero-slope limit of the Koba-Nielsen amplitudes. A systematic procedure to take the zero-slope limit *before* carrying out the integrals over the Koba-Nielsen variables will probably be a first step in this direction.

The extension of this technique to amplitudes with quarks or massive vector bosons is probably a closer goal, that we intend to pursue in the next future.

Acknowledgements

It is a pleasure to thank the organizers of this beautiful Conference and the INFN for their invitation and their support. I would also like to thank my collaborators, S. Parke and Z. Xu, for sharing with me their wisdom on the subject. I enjoyed many discussions on this topic with E. Eichten, K. Ellis, P. Marchesini and T. Taylor.

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